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Transformations for a generalized variable-coefficient nonlinear Schrödinger model from plasma physics, arterial mechanics and optical fibers with symbolic computation

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Abstract. Describing space and laboratory plasmas, arterial mechanics and optical fibers, a generalized variable-coefficient nonlinear Schrödinger model is hereby under investigation. Four transformations have been constructed from such a model to the known standard and cylindrical nonlinear Schrödinger equations with the relevant constraints on the variable coefficients presented. Symbolic computation is performed. Specialities of those transformations are discussed. Analytic solutions of such a generalized variable-coefficient model can be obtained via those transformations from the analytic solutions of the standard and cylindrical ones.

PACS. 02.70.Wz Symbolic computation (computer algebra) – 05.45.Yv Solitons – 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) – 42.65.Sf Dynamics of nonlinear optical systems; optical instabilities, optical chaos and complexity, and optical spatio-temporal dynamics – 87.19.Uv Haemodynamics, pneumodynamics

Rapid Note

Among the most important models of modern nonlinear sciences are the variable-coefficient nonlinear Schrödinger-typed ones, which describe such situations more realistically than their constant-coefficient counterparts, in plasma physics, arterial mechanics and long-distance optical communications, as seen, e.g., in references [1–18]. We hereby consider the following generalized variable-coefficient nonlinear Schrödinger model,

$$i\,u_t + k(t)\,u_{xx} + l(t)\,|\,u\,|^2\,u = -i\,\Gamma(t)\,u, \tag{1}$$

where u is a complex function of $(x,t)\in R^2$, while $k(t)$, $l(t)$ and $\Gamma(t)$ are all real functions. Equation (1) can be equivalently expressed as

$$i\,\psi_\tau + i\,\lambda(\tau)\,\psi_\xi + k(\tau)\,\psi_{\xi\xi} + l(\tau)\,\psi\,|\,\psi\,|^2 = -i\,\Gamma(\tau)\,\psi, \tag{2}$$

with the transformation [19],

$$t \longleftrightarrow \tau,\;\; x \longleftrightarrow \xi - \int \lambda(\tau)\,d\tau,\;\; u(x,t) \longleftrightarrow \psi(\xi,\tau), \tag{3}$$

where $\lambda(\tau)$ is also a real function.

In space/laboratory plasmas, fluid dynamics and optical fibers, special cases of equation (1) or (2) can be seen as follows:

(1) Space/laboratory dusty plasmas have attracted a great deal of interest [20,21]. Equation (1) can reduce to the *cylindrical and spherical geometry-modified nonlinear Schrödinger model for the dust-acoustic envelope solitary waves* [5],

$$i\;\phi_\tau - P\,\phi_{\xi\xi} - Q\,\phi\,|\,\phi\,|^2 + i\,\frac{m}{2\,\tau}\,\phi = 0, \tag{4}$$

where τ and ξ are the stretched time and radial coordinates, ϕ represents the electrostatic wave potential, $m=1$ or 2, while P and Q are the constants for plasma system [5].

(2) For the dispersion-managed optical fibers and soliton lasers, equation (1) is seen as the *generalized nonlinear Schrödinger model with varying/distributed coefficients* [1,3,6] (and references therein),

$$i\Psi_Z \pm \frac{D(Z)}{2}\Psi_{XX} + N(Z)\Psi\,|\,\Psi\,|^2 = -i\,\gamma_0\,\Psi + i\,\gamma(Z)\,\Psi, \tag{5}$$

where $D(Z)$, $N(Z)$ and $\gamma(Z)$ are respectively the dispersion, nonlinearity and amplification.

(3) In certain inhomogeneous optical fibers, a special case of equation (1) is written as

$$i\,A_z + d(z)\,A_{tt} + c(z)\,A\,|\,A\,|^2 = 0, \tag{6}$$

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where t and z represent the retarded time and propagation distance, A denotes the envelope of the optical field, while the periodic functions $d(z)$ and $c(z)$ picture out the local group-velocity dispersion and variation of power due to loss and amplification. Equation (6) can be named as the *dispersion-managed nonlinear Schrödinger model* [9], or others [7,8,10].

(4) In the space/laboratory non-uniform plasmas or optical fibers with varying loss and gain, a special case of equation (1),

$$u_Z = \frac{i}{2}\, u_{TT} + i\, a^2(Z)\, |\, u\,|^2\, u, \tag{7}$$

has been used to describe the Rayleigh-Taylor instability [11,12,22], an electron-beam plasma wave [12–14,23] and an optical soliton in a dielectric fiber [15,24]. Other equivalent forms of equation (7) include those for the pulse dynamics in the dispersion-managed fibers [25] and compression effect of chirped picosecond pulses in fibers [26].

(5) Applicable in arterial mechanics are the pressure pulses in fluid-filled expansible tubes [27]. With the blood as an incompressible viscous fluid and arteries as thin-walled, tapered, prestressed elastic tubes, nonlinear waves there can be described by the *dissipative nonlinear Schrödinger model with variable coefficient* [16],

$$i\, U_\tau + \mu_1\, U_{\xi\xi} + \mu_2\, U\, |\, U\,|^2 + i\mu_3\, \Theta\, \tau\, U_\xi + i\, \mu_7\, U = 0, \tag{8}$$

where τ and ξ are the stretched coordinates from the time and axial coordinates after static deformation, U corresponds to the dynamical radial displacement upon such initial static deformation, Θ illustrates the tapering angle, μ_7 stands for the viscosity of blood, μ_3 is contributed from variable radius, μ_1 and μ_2 are the arterial-system parameters, while the case of $\mu_4 = \mu_5 = \mu_6 = 0$ is considered. Equation (8) is a special case of equation (2).

(6) For the linearly-polarized light pulses in an inhomogeneous optical fiber, equation (2) reduces to the *variable-coefficient nonlinear Schrödinger model* [17],

$$i\, \psi_Z + i v_g^{-1}\, \psi_T - \frac{1}{2}\, \beta_2(Z)\, \psi_{TT} + \gamma\, \psi\, |\psi|^2 = 0, \tag{9}$$

where T and Z are the time and distance of transmission, ψ represents the electric-field envelope, v_g is the group velocity, γ is the nonlinear coefficient, β_2(Z) is the group-velocity-dispersion coefficient function.

(7) For the pulse compression technique capable of producing high-quality 1.3-ps pulses at a repetition rate of 10 GHz, equation (1) reduces to the *nonlinear Schrödinger model expressed in a reference frame moving at the group velocity* [18],

$$u_z = \frac{\alpha(z)}{2}\, u + \frac{i\, \beta_2}{2} u_{tt} - i\, \gamma\, u\, |\, u\,|^2, \tag{10}$$

where u is the field envelope, β_2 is the group velocity dispersion and γ is the nonlinear coefficient.

In this paper, our goals are to construct some transformations from equation (1) [or equivalently, Eq. (2)] to the known equations, obtain the corresponding constraints on the variable coefficients, and present the relevant discussions. Symbolic computation [21,28] will be carried out for dealing with the coefficient functions and other analytic expressions.

We start the work with our proposal of the format for transformations:

$$u(x,t) = A(t)\, q[\, X(x,t), T(t)\,]\, e^{\,iB(x,t)}, \tag{11}$$

where $X(x,t)$, $T(t)$, $A(t)$ and $B(x,t)$ are all real functions with $A(t) \not\equiv 0$, $T'(t) \neq 0$ and $X_x(x,t) \neq 0$, while $q(X,T)$ is a complex function to be determined. The reason for $A(t)$ to be factored out of $q(X,T)$ in format (11) is the existence of the coefficient functions of t in equation (1). Substituting format (11) back into equation (1) yields

$$\begin{aligned} &i\, A\, T'\, q_T + A\, k\, X_x^2\, q_{XX} + A^3\, l\, |\, q\,|^2\, q \\ &+ i\, q\, (\, A' + A\, k\, B_{xx} + A\, \Gamma\,) - A\, q\, (\, B_t + k\, B_x^2\,) \\ &+ i\, A\, q_X\, (\, X_t + 2\, k\, B_x\, X_x\,) + A\, k\, q_X\, X_{xx} = 0, \end{aligned} \tag{12}$$

where the prime sign denotes the differentiation with respect to t. In order to reduce equation (1) to equation (15) or (22) as below, we need to turn equation (22) into a set of partial differential equations. Symbolic computation on the set leads to the following transformations as well (details ignored):

<u>Transformation A from equation (1) to equation (15):</u>

$$u^{(A)}(x,t) = \frac{c_4}{\sqrt{|\, c_1\, [\, c_0 + \int k(t)\, dt\,]\, |}}\, q[\, X(x,t), T(t)] \times e^{-\int \Gamma(t)\, dt + i\left\{ c_6 + \frac{(c_2 + c_1 x)^2}{4\, {c_1}^2 [\, c_0 + \int k(t)\, dt]} \right\}}, \tag{13}$$

with $\quad X(x,t) = c_2\, c_3 - \dfrac{c_2 + c_1\, x}{{c_1}^2\, \left[\, c_0 + \int k(t)\, dt\right]}$,

$$T(t) = c_5 - \frac{1}{{c_1}^2\, \left[\, c_0 + \int k(t)\, dt\,\right]}, \tag{14}$$

$$q(X,T) \quad \text{satisfying} \quad i\, q_T + q_{XX} \pm |\, q\,|^2\, q = 0, \tag{15}$$

$$l(t) = \pm \frac{1}{{c_4}^2 |\, c_1\,|} \frac{e^{2\int \Gamma(t) dt} k(t)}{\left| c_0 + \int k(t) dt \right|} \quad \text{as the consistency condition,} \tag{16}$$

where c_i's are all real constants with $c_1 \neq 0$ and $c_4 \neq 0$. Equation (15) is the known *standard nonlinear Schrödinger equation*, the properties and solutions of which have been investigated in great detail, as shown, e.g., in references [17] and references therein, such as its integrability by the method of Inverse Scattering.

<u>Transformation B from equation (1) to equation (15):</u>

$$u^{(B)}(x,t) = c_{10}\, e^{-\int \Gamma(t)\, dt + i\left[c_9 - \frac{c_7\, x}{2\, m_0} - \frac{{c_7}^2 \int k(t)\, dt}{4\, {m_0}^2} \right]} \times q[\, X(x,t), T(t)\,], \tag{17}$$

with $\quad X(x,t) = c_8 + m_0\, x + c_7 \displaystyle\int k(t)\, dt$,

$$T(t) = c_{11} + {m_0}^2 \int k(t)\, dt, \tag{18}$$

$q(X,T) \quad$ satisfying equation (15),

$$l(t) = \pm\left(\frac{m_0}{c_{10}}\right)^2 e^{2\int\Gamma(t)dt} k(t) \text{ as the consistency condition,} \quad (19)$$

where $m_0 \neq 0$, c_7, c_8, c_9, $c_{10} \neq 0$ and c_{11} are all real constants.

Transformation C from equation (1) to equation (22):

$$u^{(C)}(x,t) = -\frac{\sqrt{|\,c_{12}/c_1\,|}}{c_1\,\left[\,c_0 + \int k(t)\,dt\,\right]}\, q[\,X(x,t), T(t)\,] \times e^{-\int\Gamma(t)\,dt + i\left\{c_{13} + \frac{(c_2+c_1 x)^2}{4\,c_1{}^2\,[\,c_0+\int k(t)\,dt\,]}\right\}}, \quad (20)$$

$$\text{with} \quad X(x,t) = -\frac{c_2 + c_1\,x}{c_1{}^2\,\left[\,c_0 + \int k(t)\,dt\,\right]},$$

$$T(t) = -\frac{1}{c_1{}^2\,\left[\,c_0 + \int k(t)\,dt\,\right]}, \quad (21)$$

$$q(X,T) \quad \text{satisfying} \quad i\,q_T + q_{XX} \pm |\,q\,|^2\,q + i\frac{q}{2\,T} = 0, \quad (22)$$

$$l(t) = \pm\left|\frac{c_1}{c_{12}}\right| e^{2\int\Gamma(t)dt} k(t) \quad \text{as the consistency condition,} \quad (23)$$

where $c_{12} \neq 0$ and c_{13} are both real constants. Equation (22) is the known *cylindrical nonlinear Schrödinger equation*, the properties and solutions of which have been discussed in references [5,29] and references therein, such as its perturbation solutions. In equation (22), the "±" sign represents, e.g., the unstable/stable modulation of the dust acoustic waves [5].

Transformation D from equation (1) to equation (22):

$$u^{(D)}(x,t) = c_{14}\sqrt{\left|\,c_{11} + m_0{}^2\int k(t)\,dt\,\right|}\; q[X(x,t), T(t)] \times e^{-\int\Gamma(t)\,dt + i\left\{c_9 - \frac{c_7\left[\,2\,m_0\,x + c_7\int k(t)dt\right]}{4m_0{}^2}\right\}}, \quad (24)$$

$$\text{with} \quad X(x,t) = c_8 + m_0\,x + c_7\int k(t)\,dt,$$

$$T(t) = c_{11} + m_0{}^2\int k(t)\,dt, \quad (25)$$

$$q(X,T) \quad \text{satisfying} \quad \text{equation (22)},$$

$$l(t) = \pm\frac{1}{c_{14}{}^2}\,\frac{e^{2\int\Gamma(t)\,dt}\,k(t)}{\left|\frac{c_{11}}{m_0{}^2} + \int k(t)\,dt\right|} \quad \text{as the consistency condition,} \quad (26)$$

where c_{14} is real non-zero constant.

Discussions

(1) Each analytic solution for equation (15) [or (22)] can be substituted into transformations A and B (or transformations C and D) so that the corresponding analytic solutions for equation (1) [or equivalently, Eq. (2)] come out. Especially, all the solitonic structures of equations (15) and (22) can be introduced to equation (1). Some of those analytic solutions for equation (1) may be new, not seen as yet in the existing literature.

(2) It is of interest to note that equation (1) can be reducible to the same equation (15) through both transformation A with constraint (16) and transformation B with constraint (19), which are quite different. For the same token, equation (1) can be reducible to the same equation (22) through both transformation C with constraint (23) and transformation D with constraint (26). By the way, the constraint in reference [30] is a special case of constraint (16) when $\Gamma(t) \equiv 0$.

(3) It is clear that constraints (16) and (26) are the same in the sense that c_0, c_1, c_4, c_{11}, c_{14} and m_0 are all arbitrary, non-zero, real constants. Hence, when the variable coefficients in equation (1) satisfy (the same) constraint (16) or (26), equation (1) can be transformed either into equation (15) or equation (22). Similarly, constraints (19) and (23) are the same in the sense that c_1, c_{10}, c_{12} and m_0 are all arbitrary, non-zero, real constants. When the variable coefficients in equation (1) satisfy (the same) constraint (19) or (23), equation (1) can be transformed either into equation (15) or equation (22) as well.

(4) In virtue of transformation (3), the fields $u(x,t)$ and $\psi(\xi,\tau)$ are mutually convertible. An observer in the (ξ,τ) space-time makes sure that the field $\psi(\xi,\tau)$ is clearly affected by $\lambda(\tau)$ which is contributed from the radius variation of an artery, group velocity of an optical-fiber light pulse, or another reason. On the other hand, another observer in the (x,t) space-time finds no sign of $\lambda(t)$ at all. This is because of the "absorption" during the mapping of (ξ,τ) onto (x,t).

(5) In illustration, we present transformation A from equation (4) of space/laboratory dusty plasmas to equation (15), with $m = 1$ and $t > 0$:

$$\phi^{(A)}(\xi,\tau) = \frac{c_4}{\sqrt{|\,c_1\,P\,|\,\tau}}\, e^{i\left[\,c_6 - \frac{(c_2+c_1\,\xi)^2}{4\,c_1{}^2\,P\,\tau}\,\right]} \times q\Big[c_2\,c_3 + \frac{c_2 + c_1\,\xi}{c_1{}^2\,P\,\tau}, c_5 + \frac{1}{c_1{}^2\,P\,\tau}\Big], \quad (27)$$

with q satisfying equation (15),

$$Q = \pm\frac{P}{c_4{}^2\,|\,c_1\,P\,|} \quad \text{as the consistency condition,} \quad (28)$$

where $c_0 = 0$, while $c_1 \neq 0$, c_2, c_3, $c_4 \neq 0$, c_5 and c_4 remain arbitrary.

(6) Finally, we should mention a common point of transformations A-D, which is the common effect of $\Gamma(t)$ on the fields $u(x,t)$ and $\psi(\xi,\tau)$, namely,

$$u(x,t) \sim e^{-\int\Gamma(t)\,dt}\, q[\,X(x,t), T(t)\,]. \quad (29)$$

Conclusions

In this Rapid Note, we have performed symbolic computation and constructed transformations A, B, C and D for equation (1) [or equivalently, Eq. (2)], a generalized variable-coefficient nonlinear Schrödinger model from plasma physics, arterial mechanics and optical fibers. We have discussed certain specialities of those transformations, which are able to transform equation (1) into the known standard and cylindrical nonlinear Schrödinger equations, i.e., equations (15) and (22), with the relevant constraints on the variable coefficients presented. We have also pointed out that the analytic solutions of equation (1) can be obtained via those transformations from the analytic solutions of the standard and cylindrical ones.

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